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$$\therefore 1521 = 13^2 + 34^2 + 14^2 = 13^2 + 26^2 + 26^2 = 34^2 + 2^2 + 19^2 = 26^2 + 22^2 + 19^2 \\ = 26^2 + 2^2 + 29^2 = 14^2 + 22^2 + 29^2 = 14^2 + 35^2 + 10^2.$$

### III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let  $1521 = m^2$ , and  $x^2$ ,  $y^2$ , and  $z^2$  represent the three squares; then  $x^2 + y^2 + z^2 = m^2$ , and  $x^2 = m^2 - (y^2 + z^2) = m^2 - 2pm + p^2$ .  $2pm = p^2 + y^2 + z^2$ . Let  $y = tp$  and  $z = sp$ , then  $2m = p(s^2 + t^2 + 1)$ . Restoring the value of  $m$ ,  $p^2 = 78/(s^2 + t^2 + 1)$ , in which  $s$  and  $t$  may be any rational numbers. Take  $s=1$ ,  $t=1$ , then  $p=26$ ;  $x=m-p=13$ ,  $y=tp=26$  and  $z=sp=26$ , and  $13^2 + 26^2 + 26^2 = 1521$ . Take  $s=2$ , and  $t=1$ ,  $p=13$ ;  $x=26$ ,  $y=13$ ,  $z=26$ . Take  $s=3$ ,  $t=4$ , then  $p=3$ ;  $x=36$ ,  $y=12$ ,  $z=9$ . Take  $s=2$ ,  $t=3$ ,  $p=3\frac{3}{5}$ ;  $x=2\frac{3}{5}$ ,  $y=7\frac{3}{5}$ ,  $z=11\frac{3}{5}$ , and  $[(234)^2 + (78)^2 + (117)^2]/49 = 1521$ .

While this is a solution of the question read *literally*, of course, I understand that the proposer intends to call for integral numbers; but I have obtained seven integral results, only *by trial*.

Also solved by SYLVESTER ROBINS, and G. B. M. ZERR.

### 66. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find two cubic proper fractions whose product is a square proper fraction. Can a *general* solution be made?

Solution by E. L. SHERWOOD, A. M., Superintendent City Schools, West Point, Miss.; CHARLES CARROLL CROSS, Libertytown, Md.; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Let  $a^6/b^6$ ,  $c^6/d^6$  be the fractions,  $a < b$ ,  $c < d$ .

Then  $a^6c^6/b^6d^6 = (a^3c^3/b^3d^3)^2$ .

Let  $a=1$ ,  $b=2$ ,  $c=2$ ,  $d=3$ .

$\therefore a^6/b^6 = \frac{1}{64}$ ,  $c^6/d^6 = \frac{64}{27}$ ;

$\therefore a^6/b^6 = (\frac{1}{4})^3$ ,  $c^6/d^6 = (\frac{4}{3})^3$ ;  $a^6c^6/b^6d^6 = (\frac{1}{2})^2$ .

Other fractions can easily be found.

Also solved by J. H. DRUMMOND.

67. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find (1) four consecutive numbers whose sum is a square, and (2) four consecutive numbers the sum of whose squares is a square.

### I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let  $x-1$ ,  $x$ ,  $x+1$  and  $x+2$  = four consecutive integers.

(1) Then their sum  $= 4x + 2 = 2(2x + 1) = 2$  times an odd number. But this result can never be a square.  $\therefore$  The sum of four consecutive integers can not be a square.

(2) The sum of their squares  $= 4x^2 + 4x + 6 = 2[2(x^2 + x + 1) + 1] = 2$  times an odd number, which result can never be a square.  $\therefore$  The sum of the squares of four consecutive integers can not be a square.

If, however, four consecutive numbers may be considered as four fractions whose denominators are the same number and equal to 2 times a square, and

whose numerators are consecutive integers, then we are able to fulfill the *first part* of the problem.

Of any four consecutive integers we have shown that their sum is 2 *times an odd number*. Now when this odd number is a *square*, we can find four consecutive fractions whose sum is a square, by making the denominators  $=2m^2$  and the respective numerators  $=2n(n+1)-1$ ,  $2n(n+1)$ ,  $2n(n+1)+1$ , and  $2n(n+1)+2$ . Whence we have  $\{[2n(n+1)-1]/2m^2\} + \{[2n(n+1)]/2m^2\} + \{[2n(n+1)+1]/2m^2\} + \{[2n(n+1)+2]/2m^2\} = (2n+1)^2/m^2$ .

When  $n=m=1$ , we find  $\frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} = 3^2$ . When  $n=1$  and  $m=2$ , we have  $\frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} = (\frac{3}{2})^2$ . When  $n=2$  and  $m=1$ , we find  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 5^2$ , etc.

## II. Solution by CHARLES CARROLL CROSS, Libertytown, Md.

(1) Combining the consecutive numbers we find that  $1+2+3+4$ ,  $5+6+7+8$ ,  $6+7+8+9$ ,  $7+8+9+0$ , and  $0+1+2+3$  are the only combinations whose terminal figure produces the terminal figure of a square. The first and third combinations can never produce a square number, because a square number whose terminal figure is 0 is always preceded by 0. The second and last combinations cannot produce a square number, because a square number whose terminal figure is 6 is always preceded by an odd number. The fourth combination can never be a square number, because a square number whose terminal figure is 4 is always preceded by an even number. Hence (1) is incorrect.

(2) In the *Mathematical Visitor*, Vol. I, No. 5, page 156, Dr. Martin has shown that the sum of three, of four, and of five consecutive squares, cannot be a square number. Hence (2) is also incorrect. [See also *Mathematical Magazine*, Vol. II, No. 6, page 92.]

## III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

(1) Let  $x$ ,  $x+1$ ,  $x+2$ ,  $x+3$  be the numbers.

$\therefore 4x+6=a^2$ , or  $x=(a^2-6)/4$ .

$\therefore (a^2-6)/4$ ,  $(a^2-2)/4$ ,  $(a^2+2)/4$ ,  $(a^2+6)/4$  are the numbers.

(2)  $4x^2+12x=b^2-14$ .

$\therefore x=\frac{1}{4}\sqrt{(a^2-5)}-\frac{3}{2}$ , where  $(a^2-5)$  must be a square.

Let  $a=3$ , then  $-\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$ , are the numbers.

Let  $a=2\frac{1}{2}$ , then  $-\frac{7}{6}$ ,  $-\frac{1}{6}$ ,  $\frac{5}{6}$ ,  $\frac{11}{6}$  are the numbers.

And so for other values of  $a$ .

Also solved by EDWARD R. ROBBINS, and J. H. DRUMMOND.